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A Cosmological Model with Negative Constant Deceleration Parameter in Brans-Dicke Theory

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Bermann (1983) [*Nuovo Cimento B*, **74**, 182] obtained a cosmological model with the help of special law of variation for Hubble's parameter that yields constant deceleration parameter models of the universe. Here, we present Bianchi-I model with negative constant deceleration parameter in Brans-Dicke (1961) [*Phys. Rev.*, **124**, 925] theory in the presence of perfect fluid source with disordered radiation. Some physical properties of the model are also discussed.

KEY WORDS: Radiating model; constant decelration parameter; Brans-Dicke Theory.

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1. INTRODUCTION

Brans-Dicke (1961) theory of gravitation is a well known competitor of Einstein's theory. It is the simplest example of a scalar tensor theory in which the gravitational interactions is mediated by a scalar field ϕ as well as the tensor field g_{ij} of Einstein's theory. In this theory the scalar field ϕ has the dimension of the inverse of the gravitational constant. In recent years, there has been a renewed interest in Brans-Dicke theory. The latest inflationary models (Mathiazhagan and Johri, 1984), possible "graceful exist" problem (Pimental, 1997) and extended chaotic inflations (Linde, 1990) are based on Brans-Dicke theory of gravitation.

Brans-Dicke (1961) fields equations for combined scalar and tensor fields are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\,\phi_{,j} - \frac{1}{2}g_{ij}\,\phi_{,k}\phi^{,k}\right) - \phi^{-1}(\phi_{i;j} - g_{ij}\Box\phi) \quad (1)$$

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and

$$\phi = \phi_{;K}^{;K} = 8\pi\phi^{-1}(3 + 2\omega)^{-1}T \qquad (2)$$

where $G_{ij} = R_{ij} - 1/2\ g_{ij}$ R is the Einstein tensor, T_{ij} is the stress energy tensor of the matter, ω is the dimension less coupling constant and comma and semi colon denote partial and covariant differentiation respectively.

The equations of motion

$$T_{;j}^{ij} = 0 \qquad (3)$$

are consequences of the field Eqs. (1) and (2).

Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. The work of Singh and Rai (1983) gives a detailed discussion of Brans-Dicke cosmological models. In particular, spatially homogenous Bianchi models in Brans-Dicke theory in the presence of perfect fluid with or without radiation are quite important to discuss the early stages of evolution of the universe.

Bermann (1983) presented a law of variation of Hubble's parameter that yields constant declaration parameter models of the universe. Reddy and Venkateswara Rao (2001) have obtained some cosmological models with in the frame work of Saez – Ballester (1985) scalar tensor theory of gravitation with the help of special law of variations for Hubble's parameter, while Rahaman *et al.* (2004) studied some cosmological models with negative constant declaration parameter in Lyra (1951) geometry.

In this paper, we discuss homogenous axially symmetric Bianchi-I radiating cosmological model with negative constant deceleration parameter in Brans-Dicke scalar-tensor theory of gravitation.

2. FIELD EQUATIONS AND THE MODEL

We consider a homogenous, axially symmetric Bianchi type – I metric

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)(dy^2 + dz^2)$$

We take the perfect fluid energy momentum tensor given by

$$T_{ij} = (\rho + p)u_i\, u_j - pg_{ij} \qquad (5)$$

together with

$$g_{ij}\, u^i\, u^j = 1 \qquad (6)$$

Where u^i is the four velocity vector of the fluid and p and ρ are the proper pressure and energy density respectively. From Eqs. (4)–(6) the components of T_j^i in commoving coordinates are

$$T_1^1 = T_2^2 = T_3^3 = -p, \quad T_4^4 = \rho, \quad T = \rho - 3p \qquad (7)$$

Now, the Brans-Dicke field Eqs. (1) and (2) for the metric (4) with the help of Eqs. (5)–(7) can be written as

$$2\frac{A_4 B_4}{B} + \frac{B_4^2}{B^2} + \frac{\omega}{2}\frac{\phi_4^2}{\phi^2} - \phi^{-1}(\phi_{44} - \Box\phi) = 8\pi\phi^{-1}\rho \tag{8}$$

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} + \frac{\omega}{2}\frac{\phi_4^2}{\phi^2} - \phi^{-1}\left(\frac{A_4\phi_4}{A} - \Box\phi\right) = -8\pi\phi^{-1}p \tag{9}$$

$$\frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{A_{44}}{A} + \frac{\omega}{2}\frac{\phi_4^2}{\phi^2} - \phi^{-1}\left(\frac{B_4\phi_4}{B\phi} - \Box\phi\right) = -8\pi\phi^{-1}p \tag{10}$$

$$\Box\phi = \phi^{i}_{;i} = \phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) = 8\pi\phi^{-1}(3+2\omega)^{-1}(\rho - 3p) \tag{11}$$

and Eq. (3) which is a consequence of the field equations takes the form

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) = 0 \tag{12}$$

where the suffix 4 following an unknown function denotes ordinary differentiation with respect to time t. Equations (8)–(11) are four independent equations in five unknowns A, B, ρ, p and ϕ. To get a determinate solution one extra condition is needed. So we take the equation of state.

$$\rho = 3p \tag{13}$$

which represents matter distribution with disordered radiation. In this case the set of Eqs. (8)–(11) with the help of Eq. (7) reduces to

$$2\frac{A_4 B_4}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2\frac{\phi_{44}}{\phi} = 8\pi\phi^{-1}\rho \tag{14}$$

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 - \frac{A_4\phi_4}{A\phi} = -8\pi\phi^{-1}p \tag{15}$$

$$\frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{A_{44}}{A} + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 - \frac{B_4\phi_4}{B\phi} = -8\pi\phi^{-1}p \tag{16}$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right) = 0. \tag{17}$$

We solve the above set of highly non-linear equations with the help of special law of variation of Hubble's parameter, proposed by Bermann (1983), that yields constant deceleration parameter models of the universe. We consider only constant deceleration parameter model defined by

$$q = -[RR_{44}/(R_4)^2] = \text{constant} \tag{18}$$

where $R = (AB^2)^{1/3}$ is the over all scale factor.

Here the constant is taken as negative (i.e., it is an accelerating model of the universe).

The solution of Eq. (18) is

$$R = (at + b)^{1/1+q} \tag{19}$$

Where $a \neq 0$ and b are constants of integration.

This equation implies that the condition of expansion is $1 + q > 0$.

Field Eqs. (14)–(17), with the help of Eq. (19), now, admit an exact solution given by

$$\begin{aligned}
A &= (at + b)^{1/1+q} \\
B &= \left(\frac{1}{\alpha}\right)(at + b)^{1/1+q} \\
\phi &= \left(\frac{c}{a}\right)\left(\frac{q+1}{q-2}\right)(at + b)^{\frac{q-2}{q+1}} + \phi_0 \\
8\pi\rho = 8\pi(3p) &= \left[\frac{6 - (q-2)^2(w+2) + 2(q+1)}{2(1+q)(q-2)}\right](at + b)^{-\left(\frac{q+4}{q+1}\right)}
\end{aligned} \tag{20}$$

where c and ϕ_0 are constants of integration, $\alpha \neq 0$ is a constant and q not equals zero.

Bianchi type-I radiating cosmological model corresponding to the above solution can be written (through a proper choice of coordinates and constants of integration) as

$$ds^2 = dT^2 - T^{2/1+q}dx^2 - \left(\frac{1}{\alpha^2}\right)T^{2/1+q}(dy^2 + dz^2). \tag{21}$$

3. SOME PHYSICAL PROPERTIES

The model (21) represents an exact, radiating cosmological model with a negative constant deceleration parameter (i.e., it is an accelerating universe) in the frame-work of Brans-Dicke scalar-tensor theory of gravitation.

The physical quantities that are important in cosmology are proper volume V^3, expansion scalar θ, shear scalar σ^2 and Hubble's parameter H and have the following expressions for the model given by Eq. (21) :

$$V^3 = \frac{T^{3/1+q}}{\alpha^2}$$

$$\theta = \frac{1}{3}u^i_{;i} = \frac{1}{(1+q)T}$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{6}\frac{1}{[(1+q)T]^2}$$

$$H = R_4/R = \frac{1}{(1+q)T}.$$

The expressions for the pressure p, energy density ρ and the scalar field ϕ in the model are given by

$$8\pi\rho = 8\pi(3p) = \left[\frac{2(q+1) - (q-2)^2(w+2) - 6}{2(1+q)(q-2)T^{\left(\frac{q+4}{q+1}\right)}}\right] \tag{23}$$

$$\phi = \left(\frac{q+1}{q-2}\right)T^{\left(\frac{q-2}{q+1}\right)}.$$

From Eqs. (22) and (23) one can observe that at the initial point $T = 0$, all the physical quantities diverge, while the Brans-Dicke scalar field ϕ has no initial singularity. Thus the universe starts with an infinite rate of expansion and measure of anisotropy. So this is in accordance with the big band model. Also, as $T \to \infty$, the proper volume becomes infinitely large and the other physical quantities such as density, pressure shear etc. become insignificant. In this particular case the scalar field becomes infinitely large and the shear tends to zero faster than the expansion.

Also, since $\underset{T\to\infty}{Lt}\,(\frac{\sigma^2}{\theta^2}) \neq 0$, the model does not approach isotropy. However, when $\alpha = 1$, the model becomes spatially isotropic.

In this model, particle horizon exists because

$$\int_{T_0}^{T} dT/V(T) = \left(\frac{q+1}{q}\right)[T^{q/1+q}]^T_{To} \tag{24}$$

is a convergent integral.

4. CONCLUSIONS

In this paper, we have considered Brans-Dicke (1961) field equations in the presence of perfect fluid with disordered radiation for a spatially homogenous and anisotropic Bianchi type-I space-time. In solving the field equations we have used the special law of variation of Hubble's parameter proposed by Bermann (1983). The cosmological model obtained represents a radiating universe in Brans-Dicke theory of gravitation.

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